RF-3609/200

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M. A. / M. Sc. (Part - II) Examination

April / May – 2010

Mathematics : Paper No. 503

(Advanced Linear Algebra)

Time : 3 Hours] [Total Marks : **70**

Instructions :

(1)

નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fillup strictly the details of ☛ signs on your answer book.

Name of the Examination :

☛ **M. A. / M. SC. (PART - 2)**

Name of the Subject :

☛ **MATHEMATICS - 503**

☛ Subject Code No. : **3 6 0 9** ☛ Section No. (1, 2,.....) : **NIL**

Seat No. :

Student's Signature

(2) Answer all questions.
(3) All questions carry equal marks.
(4) Follow usual notations.

Q1. a. Define Rank and Nullity of a matrix. Also if T: V→W is a linear operator then r(T) + n(T) = dim(V).

b. Let V = C(R) and W = C'(R) and define J:V→W by $J(f(x)) = \int_0^x f(t)dt$. Determine whether J is one-one or onto. Find the range of J.

c. Let $A=\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -1\\ 1 & -1 & 5\end{bmatrix}$. Find the rank and nullity of A.

OR

Q1. a. Let V=P_2 and T: V→V be defined by $(Tp)(x) = \frac{1}{2}\int_{-1}^{1}[15t^2 + 3tx + 6x^2]p(t)dt$. Construct T^{-1} and compute $T^{-1}(t^2)$. Find [T].

b. Given f is continuous on x ≥ 0. Define the numbers $x_k = \int_0^{k+1} f(t)dt$, k = 0,1,2...Is it possible to recover f from these numbers?

c. Define matrix of an operator. Also if T: $F^n \to F^m$ is a linear operator then, T(x) = Ax for all x ϵ F^n where A=[T(δ_1). T(δ_2)... T(δ_n)].

Q2. a. Let $v_i(t) = t^i$; $i = 0,1,2,3$ and let $\langle x, y\rangle = \int_0^{\infty} x(t)\overline{y(t)}\, e^{-t} dt$. Apply Gram-Schmidt process to find orthogonal polynomials with respect to the given inner product.

b. Find a linear combination s of $v_1 = (1,0,-2,1)^T$, $v_2 = (2,1,-2,0)^T$ and $v_3 = (1,-1,-1,3)^T$ for which $\|f - s\|$ is minimal, where $f = (2,3,2,2)^T$ and inner product is standard.

c. Define approximation and projection problem.

OR

Q2. a. Find the vector in $M = \text{span}\,\{E_1, E_2, E_3\}$ nearest to $f = (2,-2,-2,6)^T$ where $E_1 = (1,1,1,1)^T, E_2 = (1,-1,0,0)^T$ and $E_3 = (1,1,0,-2)^T$. Also find the distance from f to M.

b. Find the QR factorization of $\begin{bmatrix} 2 & 2 & 2 \\ 2 & -4 & 4 \\ 2 & 2 & -2 \\ 2 & -4 & 0 \end{bmatrix}$.

c. Define a projection operator. Also for a finite dimensional subspace of an inner product space V prove that the projection operator $\prod = \prod_M : V \to V$ exists and is linear.

Q3. a. Define algebraic and geometric multiplicities of an eigen value λ. Find algebraic and geometric multiplicities of elements of the spectrum of matrix $A = \begin{bmatrix} 4 & -3 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

b. Define diagonalizable operator. Prove that a linear operator on an n-dimensional space and having n distinct eigen values is always diagonalizable.

c. Find P so that $P^{-1}AP$ is diagonal matrix, where $A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$.

OR

Q3. a. Let A=$\begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$. Find (i) $(A - I)^{2000}$, (ii) e^A and (iii) $(3A - 5I)(I + 4A - 4A^2 + A^3)^{-1}$.

b. State and prove Gershgorin's first theorem.

c. Bound the eigen values of A=$\begin{bmatrix} 0 & 2 & -1 \\ 1 & 3 & 0 \\ 2 & 1 & 2 \end{bmatrix}$.

Q4. a. Let $T:\mathbb{P}_2 \to \mathbb{P}_2$ be defined by
$(Tp)(x) = \frac{3}{8}\int_{-1}^{1}[1 + 4(x + t) + 5(x^2 + t^2) - 15x^2t^2]\,p(t)dt$. Find $\sqrt{3T + 7I}$ where $h(t) = (t + 1)^2$.

b. Define characteristic polynomial and characteristic equations. Also if A is $n \times n$ matrix, then $P(\lambda) = |A - \lambda I|$ is a polynomial in λ of degree 'n' of the form
$P(\lambda) = (-1)^n\lambda^n + Tr(-1)^{n-1}\lambda^{n-1} \pm \cdots + \prod_{i=1}^{n}\lambda_i$. Furthermore
$P(\lambda) = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \ldots (\lambda_n - \lambda)$ and $Tr = \sum_{i=1}^{n}\lambda_i$ and $|A| = \prod_{i=1}^{n}\lambda_i$.

c. Define $T:\Pi_n \longrightarrow \Pi_n$ by $T(f) = f''$. Find the eigen values and eigen vectors and eigen spaces of T.

OR

Q4. a. Let $V=\prod_1$ and define T by $(Ty)(x) = y'\left(x + \frac{\pi}{4}\right)$.

b. Estimate the eigen values of $A=\begin{bmatrix} 2 & 1 & 0 \\ -1 & 4 & 1 \\ 2 & -1 & 10 \end{bmatrix}$ using Gershgorin's second theorem. Draw the discs.

c. If $D=\text{diag}(d_1, d_2, \ldots, d_n)$ and if all $f(d_i)$ are defined then
$f(D) = diag(f(d_1), f(d_2), \ldots, f(d_n))$.

Q5. a. State and prove Courant Fischer theorem.

b. Define Gram matrix. If $T:V \to V$ is linear, $\dim(V)=\infty$ and $\langle , \rangle$ is an inner product on V, then prove that the adjoint of T on V with respect to $\langle , \rangle$ exists and is unique. Further, if $B = \{e_1, e_2, \ldots, e_n\}$ is an arbitrary basis for V, and G is the Gram matrix of the inner product with respect to β, then prove that $\left[T^*\right]_\beta = G^{-1}\left[T\right]_\beta^H G$.

c. State and prove the interlacing theorem.

OR

Q5. a. Define Rayleigh quotient of an operator T. If $T:V \to \upsilon$ is Hermitian with respect to $\langle , \rangle$, $\dim(\upsilon) < \infty$, $\lambda_1 < \lambda_2 < \ldots < \lambda_r$ are distinct eigen values of T and $R(x)$ is the Rayleigh quotient of T, then prove that

(1) $R(x) = \lambda_i$ if $0 \neq x \in M(\lambda_i)$

(2) $R(x) = \lambda_1$ iff x is an eigen vector associated with λ_1.

b. State and prove the Extended Minimum Principle theorem.

c. Write the algorithm for Power method and Inverse power method.
